

# Genetic algorithm solution to the market-based transmission expansion planning problem

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The restructuring and deregulation has exposed the transmission planner to new objectives and uncertainties. As a result, new criteria and approaches are needed for transmission expansion planning in deregulated electricity markets. This paper proposes a new market-based approach for transmission planning. For the solution of this new market-based transmission expansion problem, a genetic algorithm model is proposed. Results from the application of the proposed method on the IEEE 30 bus test system demonstrate the feasibility and practicality of this approach.

(Received March 13, 2008; accepted May 5, 2008)

**Keywords:** Competitive electric markets, Transmission expansion planning, Genetic algorithm, Reference network

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## 1. Introduction

In regulated electricity markets, the transmission expansion planning (TEP) problem consists of minimizing the investment costs in new transmission lines, subject to operational constraints, to meet the power system requirements for a future demand and a future generation configuration. The TEP problem in regulated electricity markets has been addressed by mathematical optimization as well as by heuristic models. Mathematical optimization models for TEP problem include linear programming, dynamic programming, nonlinear programming, mixed integer programming, Bender's decomposition, and hierarchical decomposition [1]. Heuristic models for the solution of TEP problem include sensitivity analysis, game theory, simulated annealing, expert systems, fuzzy set theory, and genetic algorithms [1].

There are two main differences between planning in regulated and deregulated electricity markets form the point of view of the transmission planner: 1) the objectives of TEP in deregulated power systems differ from those of the regulated ones, and 2) the uncertainties in deregulated power systems are much more than in regulated ones.

The main objective of TEP in deregulated power systems is to provide a nondiscriminatory and competitive environment for all stakeholders, while maintaining power system reliability. TEP affects the interests of market participants unequally and this should be considered in transmission planning. The TEP problem in deregulated electricity markets has been addressed by probabilistic and stochastic methods. Probabilistic methods for the solution of TEP problem include probabilistic reliability criteria method, market simulation, and risk assessment [2], [3]. Stochastic methods for the solution of TEP problem include game theory and fuzzy set theory [1].

This paper proposes a general formulation of the transmission expansion problem in deregulated market

environment. The main purpose of this formulation is to support decisions regarding regulation, investments and pricing. The market-based transmission expansion problem is composed of two interrelated problems: 1) the *optimum network* problem, and 2) the *reference network* subproblem that is part of the optimum network problem. The reference network subproblem requires the solution of a type of security constrained optimal power flow problem. The market-based TEP problem (optimum network problem) is a complex mixed integer non-linear programming problem. This paper proposes a genetic algorithm model for the solution of the market-based TEP problem.

The paper is organized as follows. Section 2 formulates the market-based TEP problem. Section 3 describes the solution of the reference network subproblem using an iterative algorithm based on security constrained dc power flow. The proposed genetic algorithm based methodology for the solution of the market-based TEP problem is presented in Section 4. Section 5 contains results from the application of the proposed method to IEEE 30-bus test system. Conclusions are presented in Section 6.

## 2. Problem formulation

### 2.1 Definition

The objective of the market-based TEP problem is to optimize the transmission network topology by selecting the transmission lines that should be added to an existing transmission network so as to minimize the overall generation and transmission cost, subject to generating unit and transmission network constraints. The market-based transmission expansion problem is composed of two interrelated problems: 1) the optimum network problem, and 2) the reference network subproblem that is part of the optimum network problem.

## 2.2 Reference network subproblem

A reference network is topologically identical to an existing (or expanding) transmission network, and generators and loads are unchanged. On the other hand, each transmission line has an optimal capacity. Optimal capacities of transmission lines are determined by minimizing the sum of the annual generation cost and the annuitized cost of transmission, equation (1), subject to constraints defined by equations (2) to (9). By comparing the capacities of individual lines in the optimum reference network and the initial network, the needs for new investment in transmission lines can be identified.

The objective function of the reference network subproblem is expressed as follows [4]:

$$\min_{P_{pg}, T_b} \left[ \sum_{p=1}^{np} \tau_p \cdot \sum_{g=1}^{ng} C_g \cdot P_{pg} + \sum_{b=1}^{nl} k_b \cdot l_b \cdot T_b \right] \quad (1)$$

where  $P_{pg}$  (MW) is the output of generator  $g$  during demand period  $p$ ,  $T_b$  (MW) is the capacity of transmission line  $b$ ,  $np$  is the number of demand periods,  $\tau_p$  is the duration of demand period  $p$ ,  $ng$  is the number of generators,  $C_g$  is the operating cost of generator  $g$ ,  $nl$  is the number of transmission lines,  $k_b$  is the annuitized investment cost for transmission line  $b$  in  $\$/(\text{MW} \cdot \text{km} \cdot \text{year})$ , and  $l_b$  is the length of transmission line  $b$  in km.

This optimization is constrained by Kirchhoff's current law, which requires that the total power flowing into a node must be equal to the total power flowing out of the node:

$$A^0 \cdot F_p^0 - P_p + D_p = 0 \quad \forall p = 1, \dots, np \quad (2)$$

where  $A^0$  is the node-branch incidence matrix for the intact system,  $F_p^0$  is the vector of transmission line flows for the intact system during demand period  $p$ ,  $P_p$  is the vector of nodal generations for demand period  $p$ , and  $D_p$  is the nodal demand vector for period  $p$ .

The Kirchhoff's voltage law implies the constraint (3) that relates flows and injections:

$$F_p^0 = H^0 \cdot (P_p - D_p) \quad \forall p = 1, \dots, np \quad (3)$$

where  $H^0$  is the sensitivity matrix for the intact system.

The thermal constraints on the transmission line flows have also to be satisfied:

$$-T \leq F_p^0 \leq T \quad \forall p = 1, \dots, np \quad (4)$$

where  $T$  is the vector of transmission line capacities.

It should be noted that the constraints (2) to (4) have been derived using a dc power flow formulation neglecting losses.

The constraints (2) to (4) must also be satisfied for contingencies, i.e., for credible outages of transmission and

generation facilities. As a result, the constraints (5) to (7) have also to be satisfied:

$$A^c \cdot F_p^c - P_p + D_p = 0 \quad \forall p = 1, \dots, np; \forall c = 1, \dots, nc \quad (5)$$

$$F_p^c = H^c \cdot (P_p - D_p) \quad \forall p = 1, \dots, np; \forall c = 1, \dots, nc \quad (6)$$

$$-T \leq F_p^c \leq T \quad \forall p = 1, \dots, np; \forall c = 1, \dots, nc \quad (7)$$

where  $A^c$  is the node-branch incidence matrix for contingency  $c$ ,  $F_p^c$  is the vector of transmission line flows for contingency  $c$  during demand period  $p$ ,  $H^c$  is the sensitivity matrix for contingency  $c$ , and  $nc$  is the number of contingencies.

The optimization must respect the limits on the output of the generators:

$$P^{\min} \leq P_p \leq P^{\max} \quad \forall p = 1, \dots, np \quad (8)$$

where  $P^{\min}$  is the vector of minimum nodal generations and  $P^{\max}$  is the vector of maximum nodal generations.

Since the objective of the optimization is to find the optimal thermal capacity of the lines, this variable can take any positive value:

$$T \geq 0 \quad (9)$$

## 2.3 Optimum network problem

The optimum network problem is in fact the same with the market-based transmission expansion problem. The objective of the optimum network problem is to select the new transmission lines that should be added to an existing transmission network so as to minimize the overall generation and transmission cost.

The solution of the optimum network problem can be found by considering an exhaustive list of candidate new transmission lines, and determining which transmission lines, belonging in the exhaustive list of candidate new transmission lines, should be added to an existing transmission network so as to minimize the overall generation and transmission cost. In the process of finding the solution to the optimum network problem, the reference network subproblem should be solved for every examined combination of new transmission lines. The above presentation shows that the market-based transmission expansion problem (optimum network problem) is a complex mixed integer non-linear programming problem.

## 3. Solution of the reference network subproblem

### 3.1 Data

The data of the reference network subproblem are the following:

1. Network topology.
2. Number of demand periods.
3. Duration (h) of each demand period.
4. Load (MW) at each bus during each period.
5. Minimum generation (MW) at each bus.
6. Maximum generation (MW) at each bus.
7. Operating cost of each generator. The production (operating) cost for each unit is considered a quadratic function of the unit output.
8. Marginal annuitized investment cost [ $\$/(\text{MW} \cdot \text{km} \cdot \text{year})$ ] of each transmission line.
9. Length (km) of each transmission line.
10. Number of contingencies (outages).
11. Outages of transmission and generation facilities.

### 3.2 Design variables

The values of the design variables determine the optimal solution of the reference network subproblem. These design variables are the following:

1. The vector of generation dispatch in each demand period.
2. The vector of line power flows in each demand period.
3. The vector of the optimal transmission line capacities valid in all demand periods.

### 3.3 Solution algorithm

The reference network subproblem defined in Section 2.2 is solved using the following iterative algorithm [4]:

1. Solve the optimal power flow for each demand period.
2. Study all system conditions using a dc power flow.
3. Identify the overloaded transmission lines for each system and each demand level.
4. If all transmission line flows are within limits then go to step 6, else go to step 5.
5. Add a constraint to the optimal power flow for each overloaded transmission line and then go to step 1.
6. The optimal capacities of the transmission lines are found and the algorithm terminates.

## 4. Proposed genetic algorithm solution to the optimum network problem

### 4.1 Overview of genetic algorithms

Genetic algorithms are optimization methods inspired by natural genetics and biological evolution [5], [6]. They manipulate strings of data, each of which represents a possible problem solution. These strings can be binary strings, floating-point strings, or integer strings, depending on the way the problem parameters are coded into chromosomes. The strength of each chromosome is measured using fitness values, which depend only on the value of the problem objective function for the possible

solution represented by the chromosome. The stronger strings are retained in the population and recombined with other strong strings to produce offspring. Weaker ones are gradually discarded from the population. The processing of strings and the evolution of the population of candidate solutions are performed based on probabilistic rules.

Genetic algorithms have successfully solved many complex power system optimization problems [7]–[9]. In this paper, genetic algorithms are proposed for the solution of the market-based TEP problem, which is a complex mixed integer non-linear programming problem.

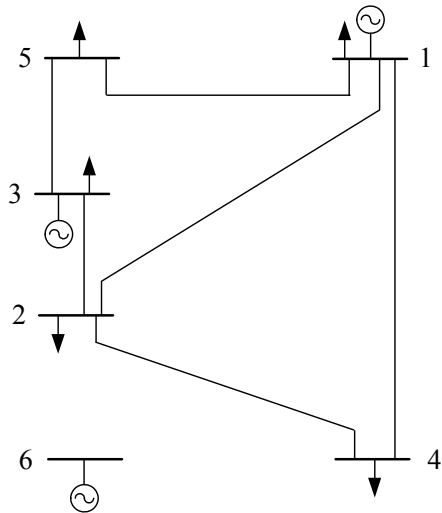
### 4.2 Overview of the Proposed Solution

The proposed genetic algorithm solution for the market-based transmission expansion problem (optimum network problem) is composed of the following steps:

1. Given the initial transmission network topology and the planned new generators, create an exhaustive list of candidate new transmission lines.
2. Create an initial population of candidate solutions. The initial population is randomly created from the exhaustive list of candidate new transmission lines.
3. While the termination criterion is not met, the genetic algorithm metaheuristic iterates over the following four phases:
  - a. Evaluation of the candidate solutions using the security constrained optimal power flow algorithm of Section 3.3.
  - b. Reproduction.
  - c. Crossover and mutation.
  - d. Creation of the next generation.
4. As soon as the termination criterion is met, the solution proposed by the genetic algorithm is the one with the minimum operating and investment cost, which simultaneously satisfies all the constraints.

In the above proposed genetic algorithm method, each chromosome represents one candidate transmission expansion scheme, i.e., a list of new transmission lines that will be added to an existing transmission network (initial network). An example of chromosome representation for transmission expansion is given in Fig. 1c. The initial network is given in Fig. 1a. Bus 6 is a new power plant to be connected to the network, so initially there is no existing transmission line between bus 6 to any bus in the network, as Fig. 1a shows. Fig. 1b presents an example of an exhaustive list of 7 candidate new transmission lines. It is important to note that if more than one new transmission lines between two buses are to be considered in the exhaustive candidate transmission expansion scheme, this has to be defined in the list of candidate transmission lines, as Fig. 1b shows, where 3 candidate new transmission lines are considered between buses 4 and 6. Fig. 1c shows one chromosome (candidate solution) of the initial population. This chromosome has been created randomly by setting zero or one for each one of the 7 candidate new transmission lines of Fig. 1b. According to this binary coding, zero means that the respective candidate new

transmission line is not considered in the candidate solution, while one means that the respective candidate new transmission line is considered in the candidate solution.



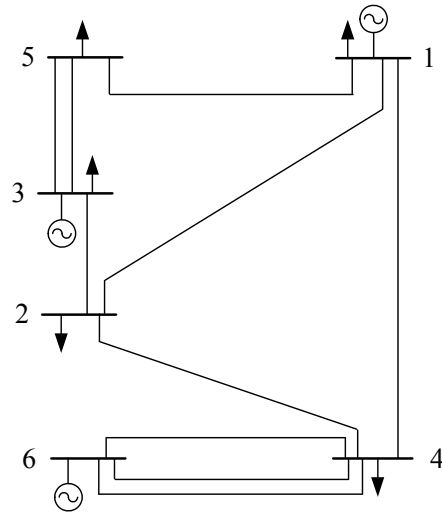
a. Initial transmission network

Line	1-2	2-4	3-5	3-6	4-6	4-6	4-6
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b. Candidate new transmission lines

Line	1-2	2-4	3-5	3-6	4-6	4-6	4-6
Chromosome	0	0	1	0	1	1	1

c. Chromosome representation of candidate transmission expansion scheme



d. Network after candidate transmission expansion

Fig. 1. An example of chromosome representation for transmission expansion.

## 5. Results and discussion

The proposed technique is extensively tested on the initial transmission network of Fig. 2 that is based on the IEEE 30 bus system [10]. As can be seen from Fig. 2, the initial transmission network is composed of 32 transmission lines and 28 buses. Bus 11 is a new power plant to be connected to the network, so initially there is no existing transmission line between bus 11 to any bus in the initial network. Bus 13 also corresponds to a new power plant.

Table 1 presents the transmission line codes of the 32 transmission lines of the initial network of Fig. 2 together with the exhaustive list of 24 candidate new transmission lines that have been considered for the solution of the transmission expansion problem for the power system of Fig. 2. By applying the proposed genetic algorithm of Section 4, it has been found that the optimum expanded transmission network has selected the 8 out of the 24 candidate new transmission lines of Table 1. Fig. 3 presents the optimum expanded transmission network for the IEEE 30-bus system. As can be seen from Fig. 3, the optimum expanded transmission network is composed of 40 transmission lines and 30 buses.

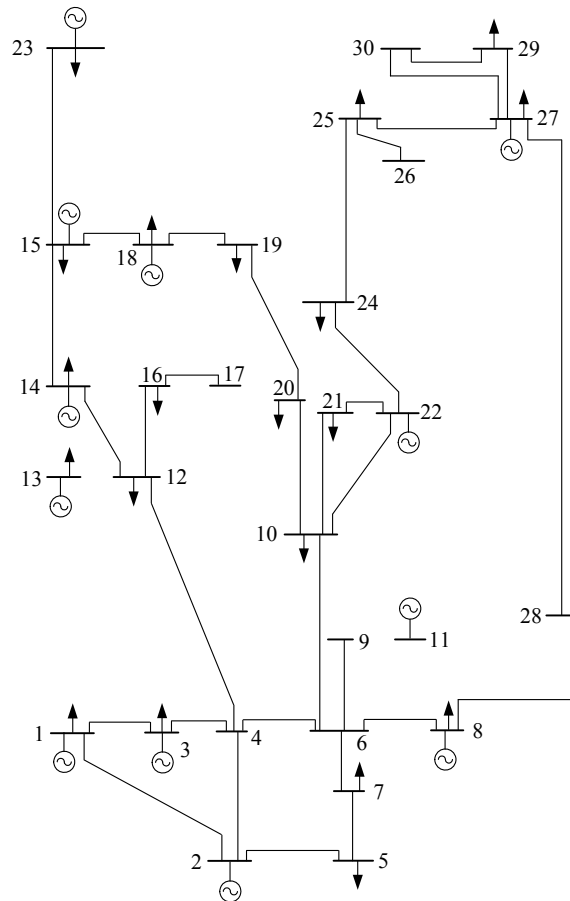


Fig. 2. Single line diagram of the initial transmission network for the IEEE 30-bus system.

Table 1. Transmission lines of the initial network of Fig. 2 (Status=I) together with the exhaustive list of candidate new transmission lines and their status: if the candidate line has been selected by the GA and belongs to the optimum network of Fig. 3, then the candidate transmission line has Status = O, otherwise it has Status=N.

Code	Line	Status	Code	Line	Status
1	1-2	I	29	25-27	I
2	1-3	I	30	27-28	I
3	2-4	I	31	27-29	I
4	2-5	I	32	27-30	I
5	3-4	I	33	2-6	O
6	4-6	I	34	6-28	O
7	4-12	I	35	9-10	O
8	5-7	I	36	9-11	O
9	6-7	I	37	10-17	O
10	6-8	I	38	12-13	O
11	6-9	I	39	12-15	O
12	6-10	I	40	23-24	O
13	8-28	I	41	5-6	N
14	10-20	I	42	6-11	N
15	10-21	I	43	10-11	N
16	10-22	I	44	10-12	N
17	12-14	I	45	10-16	N
18	12-16	I	46	10-28	N
19	14-15	I	47	11-28	N
20	15-18	I	48	12-18	N
21	15-23	I	49	13-14	N
22	16-17	I	50	13-16	N
23	18-19	I	51	15-16	N
24	19-20	I	52	16-18	N
25	21-22	I	53	17-20	N
26	22-24	I	54	19-24	N
27	24-25	I	55	20-24	N
28	25-26	I	56	23-25	N

Fig. 4 presents the capacity for pure transport in each one of the 40 transmission lines of the optimum expanded transmission network of Fig. 3 as a percentage of the optimal capacity of the respective transmission line, where the optimal capacity is the sum of two components: 1) the capacity for pure transport, and b) the capacity for security. For example, Fig. 4 shows that the transmission line with code 7, i.e., the transmission line between the buses 4 and 12 (Table 1), has 43% capacity for pure transport, while the rest 57% is its capacity for security. It can be concluded from Fig. 4 that, except for a small number of transmission lines, capacities for pure transport are well below 50% of the optimal capacities even during the period of maximum demand. This observation confirms the importance of taking security into consideration when solving the transmission expansion problem.

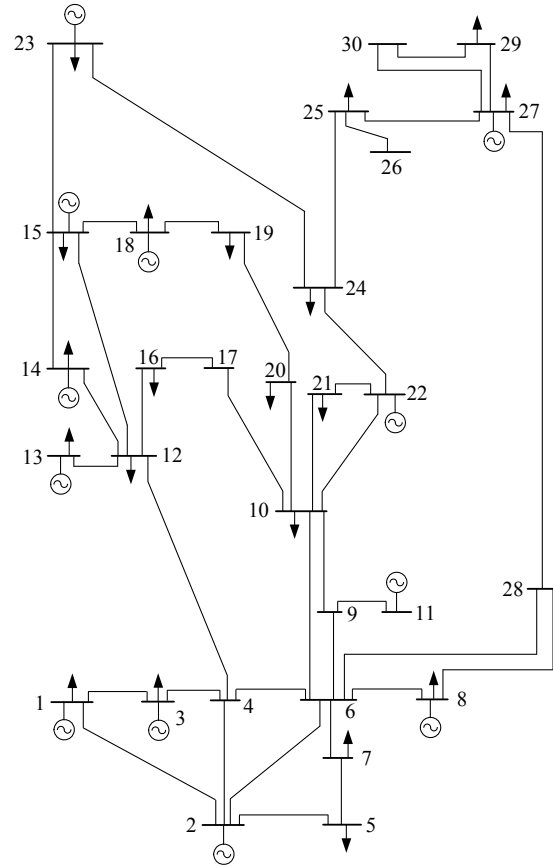


Fig. 3: Single line diagram of the optimum expanded transmission network for the IEEE 30-bus system.

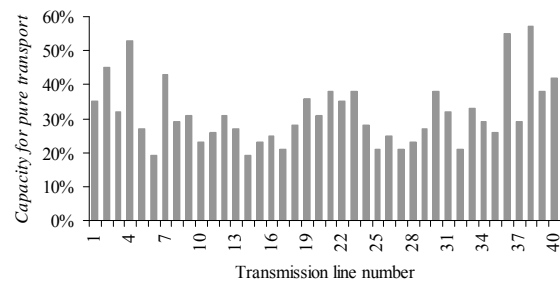


Fig. 4. Capacity for pure transport in each one of the 40 transmission lines of the optimum expanded transmission network of Fig. 3 as a percentage of the optimal capacity of the respective transmission line.

## 6. Conclusions

A general formulation of the transmission expansion problem in deregulated market environment is proposed in this paper. The main purpose of this formulation is to support decisions regarding regulation, investments and pricing. The market-based transmission expansion problem is composed of two interrelated problems: 1) the optimum

network problem, and 2) the reference network subproblem that is part of the optimum network problem. The market-based TEP problem is a complex mixed integer non-linear programming problem. This paper proposes a genetic algorithm model for the solution of the market-based TEP problem. The proposed method is applied on the IEEE 30 bus test system. Results show that, except for a small number of transmission lines, capacities for pure transport are well below 50% of the optimal capacities and this observation confirms the importance of taking security into consideration when solving the transmission expansion problem.

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